

Operator: — In Quantum Mechanics the role of Operator is very important. An Operator, Operate a function and changes the nature of function.

It means a function goes under operation by an Operator and provides another function which is different from the original function.

In Other words we can say an Operator is a symbol, giving instructions for transforming a given mathematical function into another function according to a defined rule.

Operators has no Physical meaning if it stands alone.

It is more clear by the following example.

" $\frac{\partial}{\partial x}$ " is an operator which in itself has no Physical meaning, but if put before a function, it transforms the function into its first derivative with respect to x ,

$$\text{i.e. } \frac{\partial}{\partial x} x^2 = 2x$$

Here Operator ' $\frac{\partial}{\partial x}$ ' transforms the function x^2 into another function $2x$.

If \hat{A} denotes an Operator which transforms the function $f(x)$ into another function $g(x)$,

Then we can write it as,

$$\hat{A} f(x) = g(x).$$

For every Physically measurable property of a microscopic system such as Position, momentum, Energy... etc. in Classical mechanics, there corresponds an Operator in Quantum mechanics.

For example, The operator for Linear momentum in one dimension (along x -axis i.e. P_x) is as follow

$$\begin{aligned} \hat{P}_x &= \frac{h}{2\pi i} \frac{\partial}{\partial x} \\ &= \frac{h \cdot i^2}{2\pi i \cdot i} \frac{\partial}{\partial x} \end{aligned}$$

$$= \frac{i\hbar}{2\pi i^2} \frac{\partial}{\partial x} \quad \left[i^2 = -1 \text{ \& } \frac{h}{2\pi} = \hbar \right]$$

$$\text{So, } \hat{P}_x = -i\hbar \frac{\partial}{\partial x}$$

And the Operator for angular momentum along x -axis,

$$\hat{L}_x = \frac{h}{2\pi i} \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$$

$$\hat{L}_x = -i\hbar \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$$

— x —